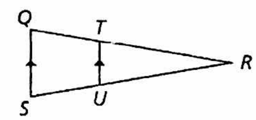
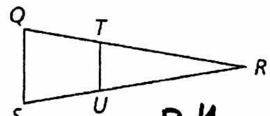
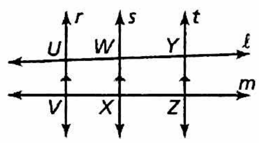
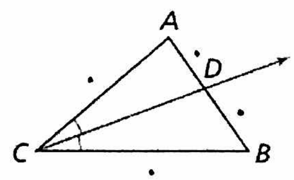
Name: Date: Period: 4

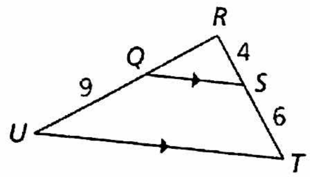
Proportionality Theorems

Lesson Objective IWBAT apply ~~[illegible]~~ proportionality thms in Δs and $\parallel$ lines cut by 2 transversals

Name	Description	Hypothesis	Conclusion
Triangle Proportionality Theorem	If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.		$\rightarrow \frac{RT}{TQ} = \frac{RU}{US}$
Converse of the Triangle Proportionality Theorem	If a line divides two sides of a triangle proportionally, then it is parallel to the third side.	 if $\frac{RT}{TQ} = \frac{RU}{US}$	$\rightarrow \overline{QS} \parallel \overline{TU}$
Three Parallel Lines Theorem	If three parallel lines intersect two transversals, then they divide the transversals proportionally.		$\rightarrow \frac{UW}{WY} = \frac{VX}{XZ}$
Triangle Angle Bisector Theorem	An angle bisector of a triangle divides the opposite side into two segments whose lengths are proportional to the lengths of the other two sides.		$\rightarrow \frac{AD}{DB} = \frac{AC}{BC}$

Example:

1. In the diagram, $\overline{QS} \parallel \overline{UT}$, RS = 4, ST = 6, and QU = 9. What is RQ?



$\frac{RS}{TS} = \frac{RQ}{QU} \rightarrow \frac{4}{6} = \frac{x}{9} \rightarrow x = 6$

Remember:

➔ The conditional and the contrapos. are logically equivalent

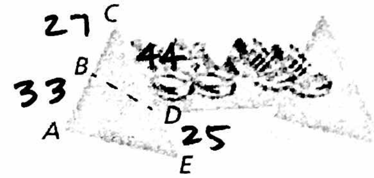
➔ The converse and the inverse are logically equivalent

The theorems above imply the following:

Contrapositive of the Triangle Proportionality Theorem	**Inverse of the Triangle Proportionality Theorem**
if $\frac{RT}{TQ} \neq \frac{RU}{US} \rightarrow \overline{TU} \nparallel \overline{QS}$	if $\overline{TU} \nparallel \overline{QS} \rightarrow \frac{RT}{TQ} \neq \frac{RU}{US}$

Examples:

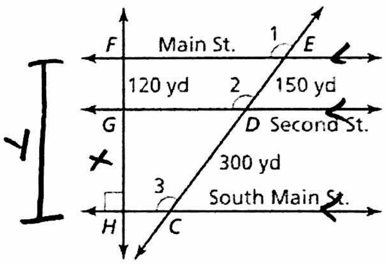
2. On the shoe rack shown, BA = 33 cm, CB = 27 cm, CD = 44 cm, and DE = 25 cm. Explain why the shelf is not parallel to the floor.



$\frac{27}{33} = \frac{44}{25} \rightarrow 675 \neq 1452$

$\overline{BD}$ does not ÷ $\overline{AC}$ and $\overline{CE}$ proportionally

3. In the diagram, $\angle 1 \cong \angle 2 \cong \angle 3$. GF = 120 yd, DE = 150 yd, and CD = 300 yd. Find the distance HF between Main Street and South Main Street.

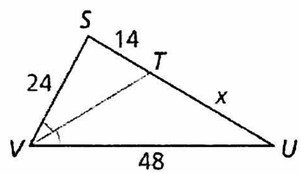


$\frac{150}{300} = \frac{120}{x} \rightarrow x = 240$

$\boxed{HF = 360 \text{ yd}}$

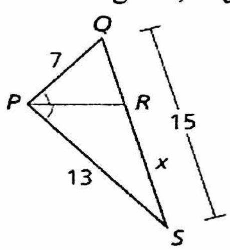
4. Find the value of x.

$\frac{ST}{TU} = \frac{VS}{VU}$



$\frac{14}{x} = \frac{24}{48} \rightarrow \boxed{x = 28}$

5. In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find RS.



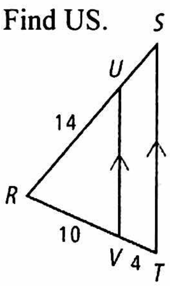
$\frac{QR}{RS} = \frac{PQ}{PS} \rightarrow \frac{15-x}{x} = \frac{7}{13}$

$\boxed{RS = 9.75}$

$7x = 195 - 13x$

$20x = 195 \longrightarrow x = 9.75$

6. Find US.

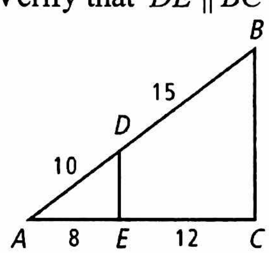


$\frac{RV}{VT} = \frac{RU}{US}$

$\frac{10}{4} = \frac{14}{x}$

$\boxed{US = 5.6}$

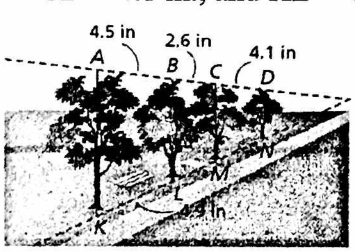
7. Verify that $\overline{DE} \parallel \overline{BC}$.



$\frac{8}{12} = \frac{10}{15}$

$120 = 120$ ✓

8. Suppose that an artist decided to make a larger sketch of the trees. In the figure, if AB = 4.5 in., BC = 2.6 in., CD = 4.1 in., and KL = 4.9 in., find LM and MN to the nearest tenth of an inch.



$\frac{4.5}{4.9} = \frac{2.6}{x}$

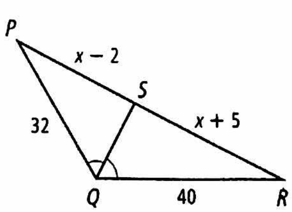
$\boxed{LM \approx 2.8 \text{ in}}$

$\frac{2.6}{4.1} = \frac{2.8}{y}$

$\boxed{MN \approx 4.5 \text{ in}}$

9. Find PS and SR.

$\frac{PS}{SR} = \frac{PQ}{QR}$



$\frac{x-2}{x+5} = \frac{\cancel{32}\,4}{\cancel{40}\,5}$

$5x - 10 = 4x + 20$

$x = 30$

$\boxed{PS = 28, \ SR = 35}$